C1.2 Conceptual Design of Small Biplanes

The golden age of the biplane is usually considered to have lasted from 1903 to 1940[3]. During this era the configuration dominated aircraft design. Although the monoplane has since surpassed the biplane, it is still a viable option for many tasks; e.g. as an aerobatic or agricultural aircraft. Both missions require rapid and responsive roll capability made possible by the compact size of its two wings. A shorter wing span is achieved by splitting the necessary wing area into two wing panels. This is beneficial in several ways. First, it substantially reduces roll damping when compared to a monoplane of the same wing area, resulting in greater roll rate. Second, the shorter wing span further reduces the moment of inertia about the roll axis, increasing the roll acceleration and reducing time required to achieve steady-state roll rate, giving the biplane great roll responsiveness. Third, the shorter wingspan reduces wing bending moments, so the wing can be made lighter and stiffer. Fourth, biplanes pack a large wing area inside a small span, allowing for reduced take-off and landing distances while eliminating the need for a heavy or complex high lift system. This also allows biplanes to operate out of unimproved landing strips with ease. Fifth, they can be designed to offer great stall characteristics by ensuring the upper wing (if a forward stagger configuration) or lower wing (if an aft stagger) stalls first. The sudden drop in lift of one of the two wings generates a nose down pitching moment, necessary for good stall recovery. Figure C1-6 shows examples of five single engine biplanes with different tail configurations, all taildraggers.

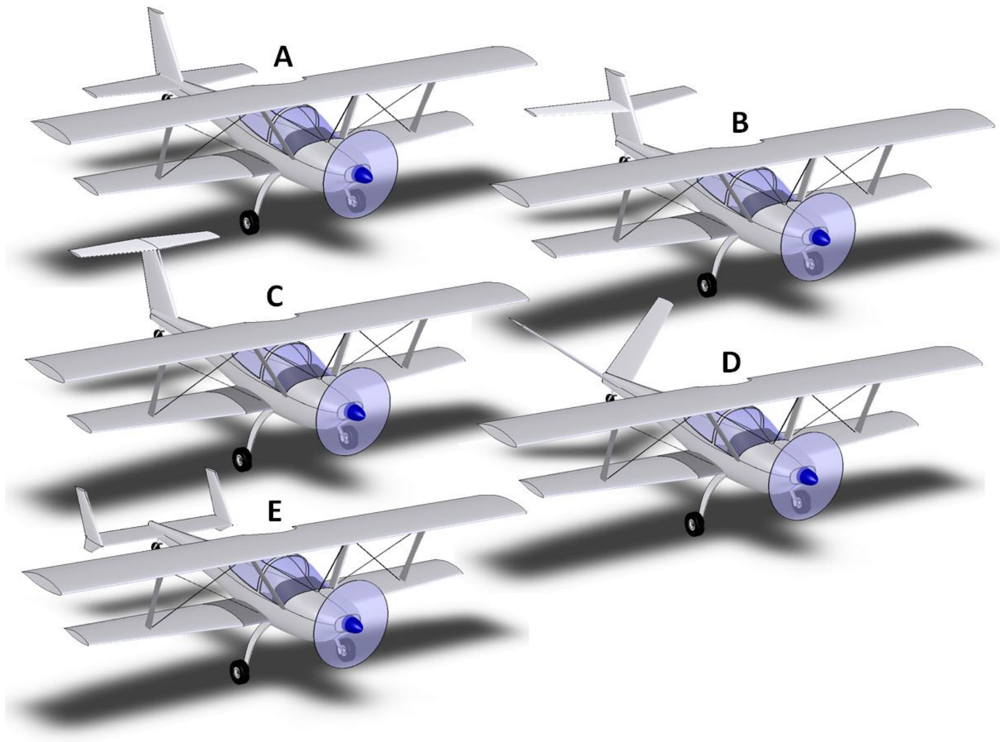

Figure C1-6: Five single-engine, taildragger biplane configurations with tractor propellers.

Among drawbacks of the configuration is the high drag of the external struts and bracing, which effectively renders the conventional biplane unsuitable for missions that involve extended range or endurance. A possible exception to this is the Griffon Aerospace Lionheart, a six-seat modernized replica of Beech's famed 1930s Staggerwing. It was designed and built in the early 2000s. It featured Natural Laminar Flow airfoils and retractable landing gear and was completely void of external struts and bracing. It was both clean and fast for a biplane, although its 450 BHP Pratt & Whitney R-945 radial engine gave it a cruising speed similar to the 300 BHP Cirrus SR22 and Cessna Corvalis (both which enjoy the safety and reduced maintenance cost of a fixed landing gear). The original Beech Staggerwing, had the upper wing aft of the lower wing, a configuration relatively rare in the history of aviation. As can be seen later, this configuration leads to a less "destructive" intereference between the two wings than the conventional arrangement, although only marginally so.

An important shortcoming of the biplane configuration is how the high pressure region of the upper wing affects the low pressure region of the lower wing. This effect can also be explained in terms of spanwise vorticity: The lower wing increases the circulation around the upper wing, while suffering from the vorticity of the upper one. This is shown in Figure C1-7. The severity of this effect depends on the relative geometry of the two wings, in particular the decalage angle (see Figure C1-9). The influence is low for zero decalage, but otherwise can be quite significant. As an example, using potential flow analysis, an average AR biplane with a 4° decalage can easily result in the upper wing generating 2-times more lift than the lower one. A 2-to-1 ratio is a very inefficient configuration - ideally, both wings should contribute equally to the total lift. This, in effect, means the lower wing is there for the ride. The biplane must make up for this inefficiency by flying at a higher AOA than a comparable monoplane. A higher AOA means increased downwash, which, in turn, means higher lift-induced drag. This is also evident by the four wingtip vortices produced by the configuration; it is indicative of a less efficient lift generation.

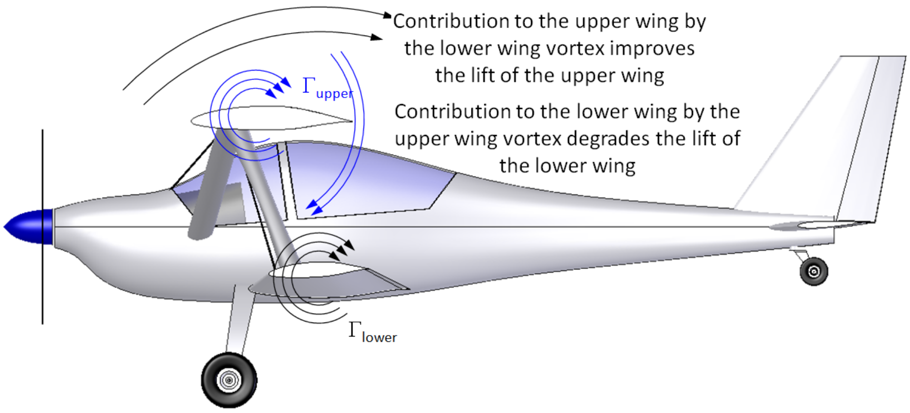

Figure C1-7: The upper wing generates more lift than the lower one due to its contribution to the upper circulation. Similarly, the upper wing vortex reduces the lift of the lower one by slowing airflow over its top surface.

Biplanes have a very shallow lift curve slope (see Figure C1-10). Therefore, they operate at high AOAs and are subject to relatively large changes in deck angle with airspeed. However, it is an advantage that it makes the airplane less susceptible to gust loading.

The Biplane as an Agricultural Aircraft

As stated earlier, the biplane configuration is well suited for agricultural aircraft. Examples of such aircraft include the Antonov An-2, Grumman Ag-Cat, Transavia PL-12 Airtruk, and the PZL M-15 Belphegor. The ideal Ag-plane must be efficient, safe and durable. In this context, efficiency refers to the airplane's ability to spray a large acreage of farmland each hour. Frequent fuel stops are a significant drawback in the operation of such aircraft, so it should feature a large fuel tank in addition to a large fertilizer tank (or *hopper* as it is referred to by operators). The airplane should also be capable of high cruising speed to allow it to be quickly ferried from one farm field to the next. The Ag-plane should be strong, reliable, and durable; capable of providing years of hard service with minimum maintenance. The ideal Ag-plane has a strong protected cockpit capable of surviving in one piece in case of even a severe accident. The cockpit should be carefully designed with pilot ergonomics and safety as priorities. For instance, pilot egress should be made easy and lightning fast. It should also feature common sense amenities like air conditioning system for added comfort; after all, it is frequently the pilot's office for up to 15 hours a day. The biplane offers an ideal solution to many of these considerations.

C1.2.1 Nomenclature for Biplanes

Figure C1-8 shows the front view of a typical biplane and the nomenclature applied to the structural arrangement between the two wings. As will be shown later, the Aspect Ratio for a biplane is obtained by dividing the square of the span of the larger wing (the upper one in Figure C1-8) by the total planform area of both wings.

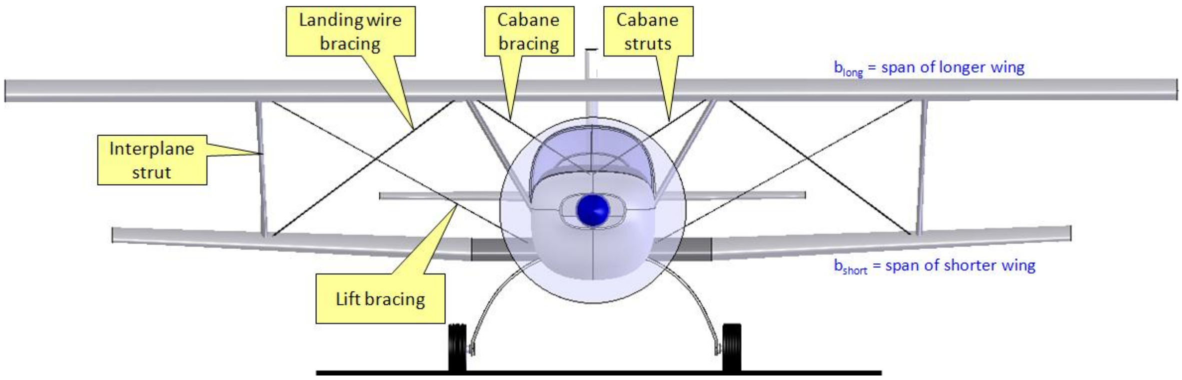

Figure C1-8: Nomenclature for the wing layout of a biplane.

C1.2.2 Various Effects that Apply to Biplanes Only

Effect of Decalage Angle

On a biplane, a *decalage angle* is the difference between the incidence angles of the top and bottom wing (see Figure C1-9). The decalage is called positive when the AOI of the lower wing is less than that of the upper wing, as shown in Figure C1-9. For one, decalage is a way to control whether the upper or lower wing stalls first. If the biplane has a positive stagger, it is desirable to ensure the upper wing stalls first. This will shift the center of lift farther aft, ensuring the airplane drops the nose gently at stall. However, the effect is more profound than that, as discussed in NACA TN-269[4]. The decalage also controls the circulation strength around the two wings. The stagger angle and gap are important characteristics because they dictate the pressure distribution between the two wings and impact the maximum lift capability (see discussion on combined effect momentarily).

Stinton[5] details that the average wing incidence for early biplanes (those that had thin undercambered airfoils) ranges from +2° to +5°. For post World War I biplanes, this value ranges from +2° to +3°. He also states that the decalage varies from 0° to +1° for both classes.

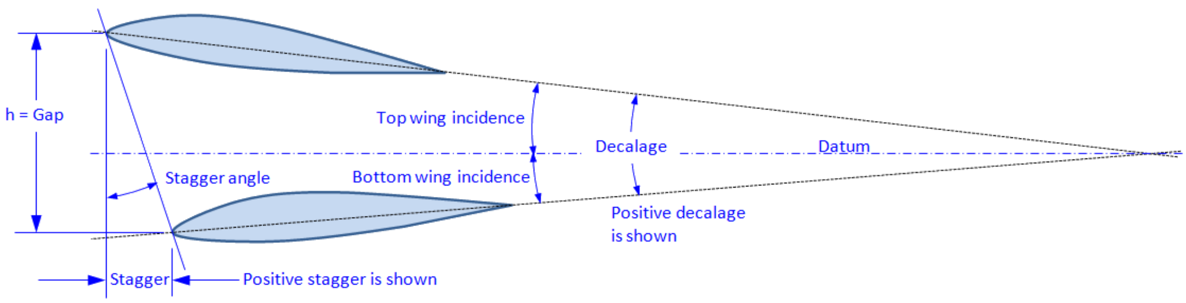

Figure C1-9: Definition of a decalage angle for a biplane.

Effect of Positive and Negative Stagger

Stagger is the relative position of the leading edges of the upper and lower wings. A positive stagger is one in which the upper wing is ahead of the lower wing. A negative stagger is the opposite. Most biplanes feature a positive stagger. The effect of stagger is investigated in NACA R-70[6], where it was concluded that a positive stagger yields a higher C_{Lmax} than a negative stagger. The greater the positive stagger, the greater the maximum lift. Additionally, positive stagger restricts the travel of the Aerodynamic Center (AC), which is helpful in stability and control. The cruise drag was marginally higher for the positive stagger, though. Stagger may be selected based on pilot visibility, as long as the designer is aware of the implications.

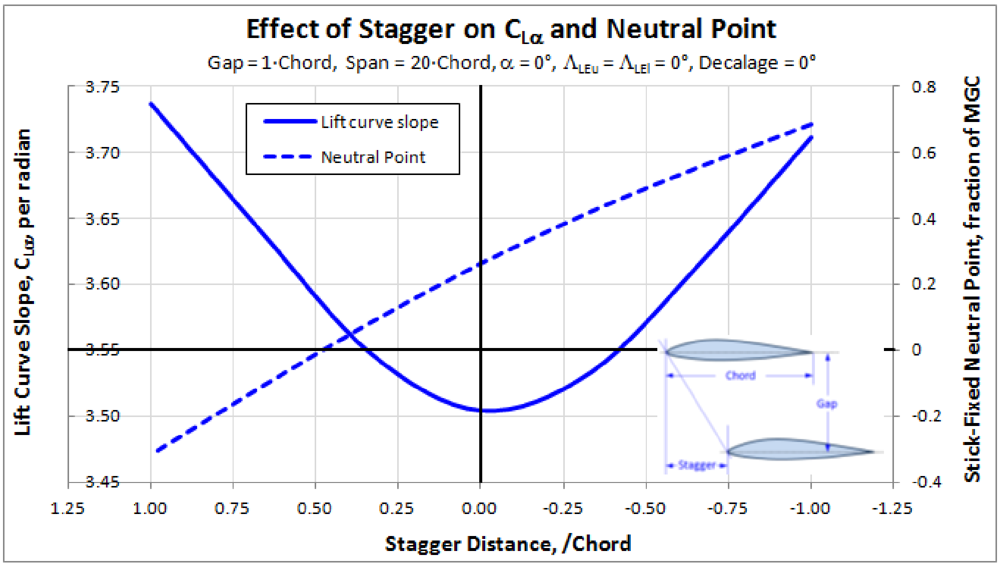

Figure C1-10: The impact of stagger on lift curve slope and stick fixed neutral point of a typical biplane wing configuration. Negative stagger means the LE of the upper wing is ahead of the lower wing LE.

Figure C1-10 shows how stagger affects the lift curve slope and the stick-fixed neutral point for a specific biplane configuration. In the graph, a positive stagger means the leading edge of the upper wing is ahead of the lower wing (note the inverted x-axis). This way, a positive stagger of 1xChord means that the trailing edge of the upper wing is directly above the leading edge of the lower one (assuming both wings have the same chord). With this in mind, consider first the lift curve slope (the solid curve). It can be seen that the maximum value of $C_{L\alpha}$ is reached at either extreme of the range evaluated. As one would expect, the minimum occurs when the upper wing is right on top of the lower one. As mentioned earlier, this is caused by the destructive interference between the high pressure region of the upper wing and the low pressure of the lower one. The lift curve slope of this configuration is approximately 93% of the extreme positive stagger (upper wing is forward of lower wing). The second observation is the change in the stick-fixed neutral point (dashed curve), here referenced to the upper wing, which moves fore and aft with it. This way, for the full positive stagger, the neutral point is approximately at 68% MGC. The MGC is considered on the upper wing. When the wings are right on top of each other, the neutral point has moved to approximately 27% MGC, and when at full positive stagger (top wing leading edge is right above the trailing edge of the lower wing), the neutral point is at -32% MGC. This way, the stagger is a tool to modify the longitudinal stability characteristics of the design.

Combined Effect of Stagger and Decalage

Figure C1-11 shows the influence of selected combinations of positive and negative stagger and decalage. Using the vortex analogy of Figure C1-7, it can be seen that the positive decalage results in the lift forces adding to form the total lift. However, the interference is destructive because it reduces the lift effectiveness of both wings (i.e. the total lift is less than it would be in its absence). The opposite holds for the negative stagger; the lift forces must be subtracted. However, the circulation direction will be additive – therefore, the magnitude of the two forces can be expected to be greater.

Figure C1-12 shows an example of some arbitrary biplane configuration, for which the two wings have an equal chord. The upper wing of the left combination has an AOI of +6° and generates a C_{Ltot} of 0.2441. The upper wing of the right combination has an AOI of -6° with a C_{Ltot} of 0.0163. Although this combination generates the least amount of total lift, the lower wing is operating at a higher C_L than the others, due to the constructive interference of the wing circulation. The rightmost configuration presents a possible aircraft configuration in which the upper wing, of a reduced chord, can be used to enhance lift on the lower one (the main wing) while acting as a possible horizontal tail or as a part of a tandem wing layout.

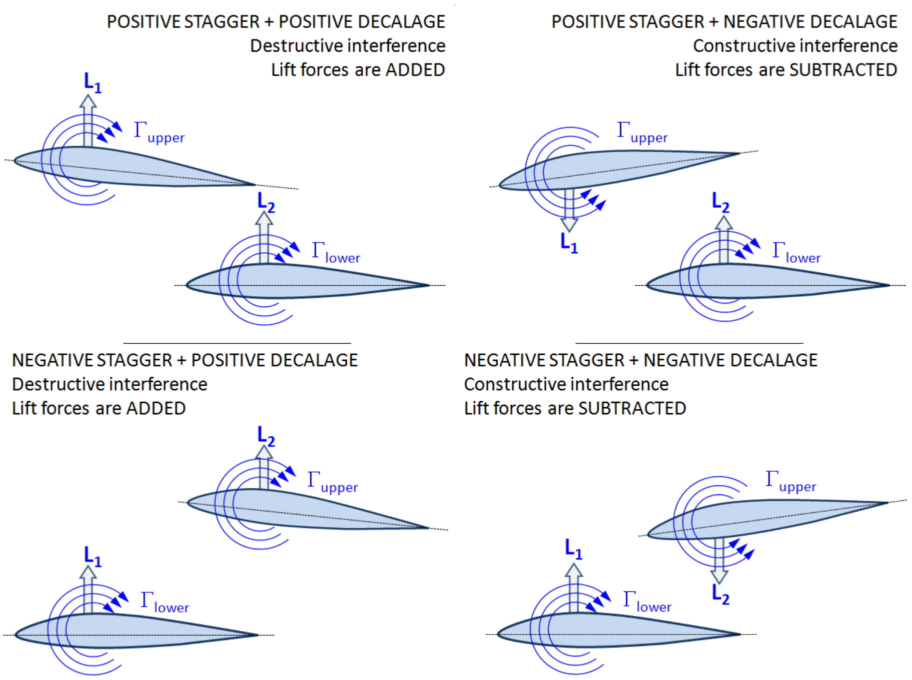

Figure C1-11: The impact of stagger on lift curve slope and stick fixed neutral point of a typical biplane wing configuration. Positive stagger means the LE of the upper wing is ahead of the lower wing LE.

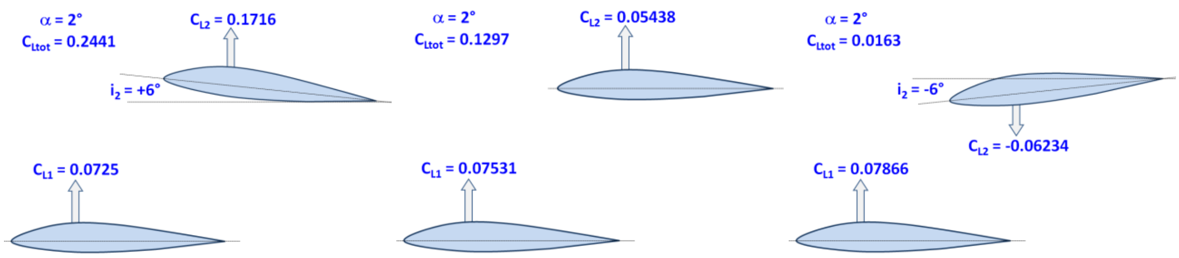

Figure C1-12: Typical results for a negative stagger for some arbitrary biplane configuration using potential flow theory. That α of the lower wing is 2° for all combinations. The left combination has a positive decalage and the right combination has a negative decalage. To generate a positive fixed CL, the rightmost configuration must operate at the largest AOA of the three.

Effect of Gap

The *gap* is the space between the upper and lower wings. A large gap will generally reduce drag by improving the flow field between the two wings. Of course, the larger the separation, the larger will be the wetted area of the support struts and bracing wires, not to mention reduced buckling strength of struts that react compressive flight loads. The effect of drag is accounted for using the formulation that follows. Buckling strength is handled during the detail design phase.

C1.2.3 Aerodynamic Properties of the Biplane Configuration

The following is a summary of how to determine important design parameters for the biplane configuration.

Biplane Aspect Ratio
The Aspect Ratio of a biplane is given by:

$$\boxed{AR_{biplane} = \frac{2b^2_{\text{larger wing}}}{S}} \tag{C1-1}$$

Equivalent Monoplane Theorem
The concept of an *equivalent monoplane* was proposed by Munk[7] in the early 1900s to help simplify analyses of biplanes. It presumes the biplane configuration can be replaced with a monoplane wing of equal wing area and lift-induced drag. It has already been stated that the biplane must generate greater downwash than a monoplane to maintain altitude. This difference can be represented as follows:

$$\frac{\varepsilon_{biplane}}{\varepsilon_{monoplane}} = 1 + \frac{\Delta\varepsilon}{\varepsilon} = 1 + \sigma \tag{C1-2}$$

Where: $\varepsilon_{\text{monoplane}}$ = Downwash by a monoplane $= \dfrac{C_L}{\pi \cdot AR \cdot e}$

$\varepsilon_{\text{biplane}}$ = Downwash by a biplane of equal weight and wing area $= \dfrac{(1+\sigma)C_L}{\pi \cdot AR \cdot e}$

σ = Biplane interference factor (to be discussed momentarily in more detail)

Munk's Span Factor, *k*
As stated above, Munk's *Equivalent Monoplane Theorem* replaces the biplane wings with a monoplane wing of equal area and lift-induced drag. This way, if the maximum wing span of the biplane is given by b, then a corresponding equivalent monoplane wingspan will be k·b, where k is called the *Munk's Span Factor*[8]. The factor k has a value of 1 for monoplanes, but for biplanes it is always larger than 1 and is a function of the following ratios:

(1) Gap ratio, which is gap/(average wingspan) = $\left(h/b_{avg}\right)$,

(2) Span ratio, which is (shorter wingspan)/(longer wingspan), $\mu = b_{short}/b_{long}$, and

(3) Area ratio, $r = S_{long}/S = \left(S - S_{short}\right)/S$.

Where: h = Gap height (see Figure C1-9)

$b_{avg} = \frac{1}{2}\left(b_{long} + b_{short}\right)$ (see Figure C1-8)

S = Total area of both wings.

S_{long} and S_{short} = Planform areas of the two wings.

The Munk's span factor is given by the following expression:

$$\boxed{k = \sqrt{\frac{2}{1+\sigma}}} \tag{C1-3}$$

Biplane Interference Factor, σ
The biplane interference factor accounts for the fact that the presence of two lifting surfaces in a close proximity will affect the resulting flow field. In other words, the upper wing affects the lower wing and vice versa. This

interaction can be measured and is represented using the factor σ. A method developed by Prandtl[9] can be used to estimate σ if both wings are of an equal span. It is valid for $0.05 \le (h/b_{avg}) \le 0.5$:

$$\boxed{\sigma = \frac{1 - 0.66(h/b_{avg})}{1.055 + 3.7(h/b_{avg})}} \tag{C1-4}$$

Diehl8 presents a graph of Prandtl's biplane interference factors for other span ratios, μ. Using surreptitious mathematical wizardry, the following expression was derived to calculate σ for other span ratios. It is valid for $0.4 \le \mu \le 1.0$ and $0.05 \le (h/b_{avg}) \le 0.5$. It provides an acceptable fit for the curves in Diehl's graph.

$$\boxed{\sigma = \left(\frac{6}{75}\right)\frac{75\mu - (28 + 20\mu)(h/b_{avg})}{6 + (29\mu - 5)(h/b_{avg})}} \tag{C1-5}$$

This is plotted in Figure C1-13.

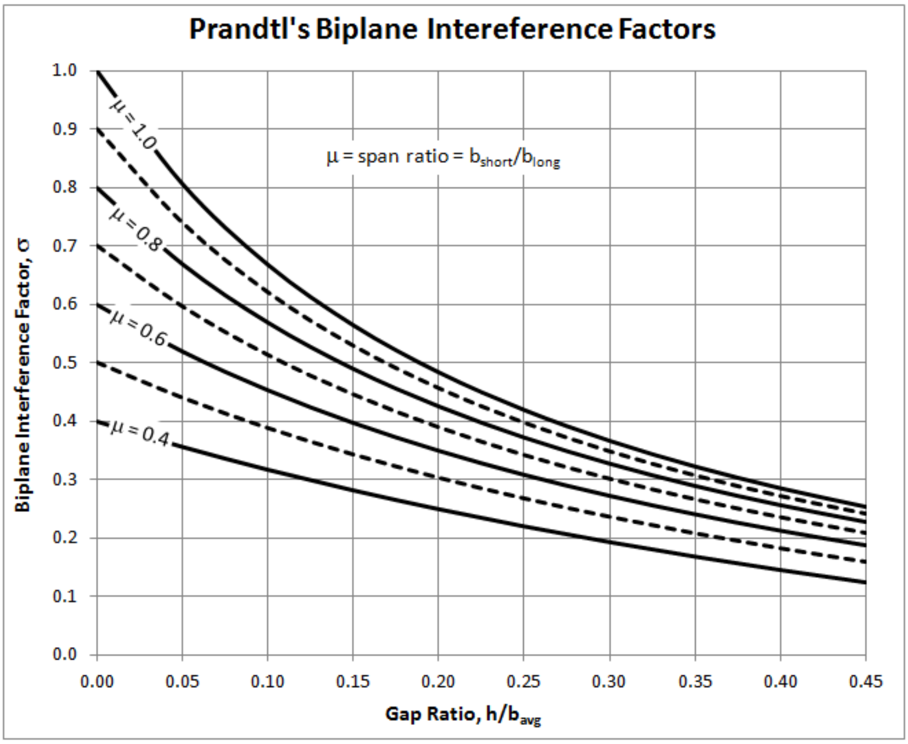

Figure C1-13: A map of Prandtl's biplane interference factors as functions of the gap and span ratios. (Based on Reference 8.)

Lift-Induced Drag of a Biplane

Once the biplane interference factor has been determined, the lift-induced drag can be determined using the following expression:

Biplane lift-induced drag:

$$\boxed{C_{Di} = \frac{C_L^2}{\pi \cdot AR \cdot e}(1 + \sigma) = \frac{SC_L^2}{\pi \cdot (2b^2) \cdot e}(1 + \sigma)} \tag{C1-6}$$

Prandtl also showed that the drag of one wing in the presence of the other is given by:

$$D_{12} = \left(\frac{\sigma}{\pi q}\right)\frac{L_{long}L_{short}}{b_{long}b_{short}} \tag{C1-7}$$

Where: q = Dynamic pressure
L_{long} and L_{short} = Lift of the two wings
b_{long} and b_{short} = Span of the two wings.

Furthermore, he determined the total lift-induced drag to be:

$$D_i = \left(\frac{1}{\pi q}\right)\left[\frac{L_{long}^2}{b_{long}^2} + 2\sigma\frac{L_{long}L_{short}}{b_{long}b_{short}} + \frac{L_{short}^2}{b_{short}^2}\right] \tag{C1-8}$$

The expression allows the geometry for minimum lift-induced drag to be determined. This happens when:

$$\frac{L_{long}}{L_{short}} = \frac{\mu - \sigma}{1/\mu - \sigma}$$

The resulting minimum lift-induced drag is thus found to be:

Minimum lift-induced drag:

$$D_i = \left(\frac{L^2}{\pi q b_{long}^2}\right)\left[\frac{1-\sigma^2}{1-2\sigma\mu+\mu^2}\right] \tag{C1-9}$$

Where: $L = L_{long} + L_{short}$ = Lift (or weight of the aircraft)

If the two wings are of different geometry, then it is traditional to assume the lift is proportional to the area ratio, r (defined earlier). This way, the following rules hold:

Wing areas: $r = \frac{S_{long}}{S} = \frac{S - S_{short}}{S} \Leftrightarrow S_{long} = rS \quad and \quad S_{short} = (1-r)S$

Wing lift: $L_{long} = rL = rW \quad and \quad L_{short} = (1-r)L = (1-r)W$

Substituting this into Equation (C1-9) leads to another helpful expression in terms of weight at condition, W, and the area ratio:

Minimum lift-induced drag:

$$D_i = \left(\frac{W^2}{\pi q b_{long}^2}\right)\left[r^2 + \frac{2\sigma}{\mu}r(1-r) + \left(\frac{1-r}{\mu}\right)^2\right] \tag{C1-10}$$

Finally, as presented by Diehl8, the Munk's span factor for this optimized configuration is then given by:

$$k = \sqrt{\frac{\mu^2}{r^2(\mu^2 - 2\mu\sigma + 1) + 2r(\mu\sigma - 1) + 1}} \tag{C1-11}$$

Diehl also presents a number of graphs, not presented here, that will help further in the design of efficient biplanes.

With these tools in hand, it is now possible to implement a reasonable performance analysis for the biplane. The foregoing formulation, and in particular the presence of the biplane interference factor, σ, reveals that even a very clean biplane will always generate more drag than a comparable monoplane. This is further compounded by a larger interference drag due to two rather than one wing, which increases C_{Dmin} as well. For this reason, L/D efficiency is not a compelling argument for such a design, but rather the other favorable properties discussed at the beginning of this section.

DERIVATION OF EQUATION (C1-3):
Lift-induced drag of the equivalent monoplane of wing span k·b is given by:

$$C_{Di} = \frac{C_L^2}{\pi \cdot AR \cdot e} = \frac{C_L^2}{\pi \cdot \left(k^2 b^2 / S\right) \cdot e} = \frac{SC_L^2}{\pi \cdot \left(k^2 b^2\right) \cdot e} \qquad \text{(i)}$$

Lift-induced drag of the biplane:

$$C_{Di} = \frac{C_L^2}{\pi \cdot AR \cdot e}(1+\sigma) = \frac{C_L^2}{\pi \cdot \left(2b^2 / S\right) \cdot e}(1+\sigma) = \frac{SC_L^2}{\pi \cdot \left(2b^2\right) \cdot e}(1+\sigma) \qquad \text{(ii)}$$

Per Munk, the lift-induced drag for the biplane and its equivalent monoplane configuration must be equal, but this allows the factor k to be determined, yielding Equation (C1-3).

QED